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EQUIVALENCE OF FORMULATIONS OF PROBLEMS WHEN MODELING FLOWS OF RHEOLOGICALLY COMPLEX MEDIA IN SCREW-SHAPED CHANNELS

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The equivalence of two formulations of problems concerning the flow of a Newtonian liquid in a screw-shaped channel of an extruder – direct and inverse (rotation of jacket) – is analyzed.

The problem of the motion of a liquid in the screw-shaped channel of an extrusion machine is traditionally formulated as an inverse problem. In this formulation the screw is stationary and the casing rotates, and the problem is ultimately reduced to flow in a rectangular channel whose upper wall moves at an angle with respect to the longitudinal axis [1-3].

A different, direct formulation of the problem is also possible [4]. In this formulation the screw rotates and the casing is stationary. The solution of the problem in this case is obtained with the help of spiral coordinates introduced in a different manner. An example of such a formulation is given in [5].

Since the problem of accurate calculation of extruders (on which, by the way, there are many papers and monographs) is important, it is useful to study the relation between the two approaches to modeling.

When analyzing the direct formulation it should first be noted that in both [4] and [5] nonorthogonal spiral coordinate systems are introduced. In [6] it is proved that the velocity vector is self-similar relative to the third (spiral) coordinate. We first show that it is impossible to introduce orthogonal coordinates in which the spiral displacement is transformed into a translation of the coordinate.

Let $S_\alpha : R^3 \to R^3$ be a spiral displacement by an angle α (if the axis of the screw is taken as the Oz axis and the Ox and Oy axes are chosen to be orthogonal to the Oz axis, then this transformation has the form: $x \to x\cos\alpha - y\sin\alpha$, $y \to x\sin\alpha + y\cos\alpha$, $z \to z + \gamma\alpha$). The trajectory of a point M is the curve $\{S_\alpha M\}_{-\infty<\alpha<\infty}$ – the spiral line. We shall show that in a neighborhood of the point M it is possible to introduce an orthogoanl coordinate system so that the trajectories of the points in a neighborhood of M would be coordinate lines.

For this we show that there does not exist a surface orthogonal to the spiral lines in a neighborhood of the point M. This is an obvious consequence of Frobenius's theorem [7]. Here we shall give a direct proof.

We write the parametric equations of the spiral line passing through the point (x^0, y^0, z^0) as follows:

$$\begin{aligned} x &= x^0\cos\alpha - y^0\sin\alpha, \\ y &= x^0\sin\alpha + y^0\cos\alpha, \quad -\infty < \alpha < \infty, \\ z &= z^0 + \gamma\alpha, \end{aligned} \qquad \begin{cases} x(0) = x^0, \\ y(0) = y^0, \\ z(0) = z^0. \end{cases}$$

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The tangent vector to this line at the point (x^0, y^0, z^0) is the vector $\{-y^0, x^0, \gamma\}$. We assume that there exists in a neighborhood of the point $M(x^0, y^0, z^0)$ a twice-differential function $\Psi(x, y, z)$ such that the surfaces of constant level $\Psi(x, y, z) = C$ are orthogonal to the spiral lines, i.e., $(\operatorname{grad}\Psi)(x, y, z) \parallel \{-y, x, \gamma\}$ in a neighborhood of M. This means that

$$\frac{\dfrac{\partial\Psi}{\partial x}}{-y} = \frac{\dfrac{\partial\Psi}{\partial y}}{x} = \frac{\dfrac{\partial\Psi}{\partial z}}{\gamma} \qquad (1)$$

for any (x, y, z) in a neighborhood of M. From here

$$\frac{\partial\Psi}{\partial y} = \frac{x}{\gamma}\frac{\partial\Psi}{\partial z}, \quad \frac{\partial\Psi}{\partial x} = -\frac{y}{\gamma}\frac{\partial\Psi}{\partial z}. \qquad (2)$$

Differentiating the first equality in Eq. (2) with respect to x and the second with respect to y

$$\frac{\partial^2\Psi}{\partial x\partial y} = \frac{1}{\gamma}\frac{\partial\Psi}{\partial z} + \frac{x}{\gamma}\frac{\partial^2\Psi}{\partial x\partial z}, \quad \frac{\partial^2\Psi}{\partial y\partial x} = -\frac{1}{\gamma}\frac{\partial\Psi}{\partial z} - \frac{y}{\gamma}\frac{\partial^2\Psi}{\partial y\partial z},$$

and then equating the mixed derivatives, we obtain

$$\frac{2}{\gamma}\frac{\partial\Psi}{\partial z} + \frac{x}{\gamma}\frac{\partial^2\Psi}{\partial x\partial z} + \frac{y}{\gamma}\frac{\partial^2\Psi}{\partial y\partial z} = 0. \qquad (3)$$

Differentiating Eq. (2) with respect to z, we obtain

$$\frac{\partial^2\Psi}{\partial z\partial y} = \frac{x}{\gamma}\frac{\partial^2\Psi}{\partial z^2}, \quad \frac{\partial^2\Psi}{\partial z\partial x} = -\frac{y}{\gamma}\frac{\partial^2\Psi}{\partial z^2}. \qquad (4)$$

Substituting Eq. (4) into Eq. (3) we obtain

$$\frac{2}{\gamma}\frac{\partial\Psi}{\partial z} - \frac{xy}{\gamma^2}\frac{\partial^2\Psi}{\partial z^2} + \frac{yx}{\gamma^2}\frac{\partial^2\Psi}{\partial z^2} = 0.$$

Hence $\partial\Psi/\partial z = 0$ and, therefore (see Eqs. (1)), $\partial\Psi/\partial y = 0$, $\partial\Psi/\partial x = 0$, which means that Ψ is constant and shows that the surfaces sought do not exist.

We denote by D_t the region occupied by the liquid at the time t; by K_t the part of the region D_t belonging to the jacket; by L_t the part of the boundary D_t belonging to the spiral; $\overline{V}(\overline{\chi}, t)$ the row vector of the flow velocity at the point $\overline{V}(\overline{\chi}, t)$ at time t. Then the boundary-value problem of the flow of an elastoviscous liquid (for the example of the Reiner–Rivlin liquid) in the spiral channel of an extrusion machine can be written as

$$\rho\left(\frac{\partial\overline{V}}{\partial t} + \overline{V}\,\overline{\nabla}\,\overline{V}\right) - \nabla^{\tau}P + \nabla^{\tau}[\varphi_1(I_2(\overline{V}))B(\overline{V}) + \varphi_2(I_2(\overline{V}))B^2(\overline{V})] = 0, \qquad (5)$$

$$S_p(\overline{\nabla}\,\overline{V}) = 0, \ \overline{\chi} \in D_t, \qquad (6)$$

with the slip boundary condition at the wall

$$-\lambda(I_2(\overline{V}))\overline{V} = pr_{N_K^{\perp}}\overline{N}_K C(\overline{V})|_{K_t}, \qquad (7)$$

$$-\lambda(I_2(\overline{V}-\overline{V}_L))(\overline{V}-\overline{V}_L) = pr_{N_L^{\perp}}\overline{N}_L C(\overline{V})|_{L_t}. \qquad (8)$$

Here $B(\overline{V}) = \overline{\nabla}\overline{V} + (\overline{\nabla}\overline{V})^T$; $I_2(\overline{V}) = S_p B^2(\overline{V})$; $\overline{N}_K(\overline{\chi})$ is the unit inner vector normal to K_t at the point $\overline{\chi} \in K_t$; $\overline{N}_L(\overline{\chi})$ is the unit outer vector normal to L_t at the point $\chi \in L_t$; $\overline{V}_L = \overline{\omega} \times \overline{\chi}$ is the rotational velocity of the point $\chi \in L_t$; T designates transposition; $pr_{N^{\perp}}$ is the orthogonal projection on a plane perpendicular to $\overline{N}$; and

$$C(\overline{V}) = \varphi_1(I_2(\overline{V}))B(\overline{V}) + \varphi_2(I_2(\overline{V}))B^2(\overline{V}).$$

Equation (5) of the system (5)-(8) is the equation of motion and Eq. (6) is the equation of continuity. The boundary conditions (7) and (8) are based on the assumption that on the channel walls the normal component of the relative velocity is equal to zero while the tangential component is proportional to the tangential component of the viscosity force. The slipping factor λ depends, as usual [3], only on $I_2(\overline{V})$.

We write $\overline{W} = \overline{V} - \overline{V}_L$ and $\overline{V} = \overline{W} + \overline{V}_L$. Orienting the Oz axis, as usual, along the axis of the screw and the Ox and Oy axes orthogonally to Oz, it is obvious that $\overline{w} = (0, 0, w)$, $\overline{V}_L = \overline{w} \times \overline{\chi} = (-wy, wx, 0)$,

$$\overline{\nabla}\,\overline{V}_L = \begin{pmatrix} 0 & w & 0 \\ -w & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ \overline{\nabla}\,\overline{V}_L + (\overline{\nabla}\,\overline{V}_L)^{\tau} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Hence $B(\overline{V}_L) = 0$, $B^2(\overline{V}_L) = 0$, $I_2(\overline{V}_L) = S_p B^2(\overline{V}_L) = 0$, $S_p\ (\overline{\nabla}\overline{V}_L) = 0$. It is also obvious that $\partial V_L/\partial t = (0, 0, 0)$.

We set $P_1 = -\rho w^2/2\ (x^2 + y^2)$, $P = P_1 + P_2$. Substituting $\overline{V} = \overline{W} + \overline{V}_L$ into the system and taking into account the equalities obtained above, we have:

$$\rho\left(\frac{\partial \overline{W}}{\partial t} + \overline{W}\overline{\nabla}\overline{W} + \overline{V}_L\overline{\nabla}\overline{W} + \overline{W}\overline{\nabla}\overline{V}_L + \overline{V}_L\overline{\nabla}\overline{V}_L\right) - \nabla^{\tau} P_1 - \tag{9}$$

$$- \nabla^{\tau} P_2 + \nabla^{\tau}\left[\varphi_1 (I_2(\overline{W}))\, B(\overline{W}) + \varphi_2 (I_2(\overline{W}))\, B^2(\overline{W})\right] = 0,$$

$$S_p(\overline{\nabla}\overline{W}) = 0,\ \overline{\chi} \in D_t, \tag{10}$$

$$-\lambda(I_2(\overline{W}))(\overline{W} + \overline{V}_L) = pr_{N_K^{\perp}} \overline{N}_K\, C(\overline{W})|_{K_t}, \tag{11}$$

$$-\lambda(I_2(\overline{W}))\,\overline{W} = pr_{N_L^{\perp}} \overline{N}_L\, C(\overline{W})|_{L_t}. \tag{12}$$

Here we employed the facts that $B(\overline{W} + \overline{V}_L) = B(\overline{W}) + B(\overline{V}_L) = B(\overline{W})$, $B^2(\overline{W} + \overline{V}_L) = B^2(\overline{W})$, $I_2(\overline{W} + \overline{V}_L) = I_2(\overline{W})$. Hence $C(\overline{W} + \overline{V}_L) = C(\overline{W})$.

We chose $P_1 = -\rho\omega^2/2\ (x^2 + y^2)$ so that $\rho\overline{V}_L\overline{\nabla}\overline{V}_L - \nabla^T P_1 = 0$. Indeed,

$$\rho\overline{V}_L\overline{\nabla}\,\overline{V}_L - \nabla^{\tau} P_1 = \rho(-wy,\ \ wx,\ 0)\begin{pmatrix} 0 & 0 & 0 \\ -w & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + (\rho w^2 x,\ \rho w^2 y,\ 0) =$$

$$= (-\rho w^2 x,\ -\rho w^2 y,\ 0) + (\rho w^2 x,\ \rho w^2 y,\ 0) = (0,\ 0,\ 0)$$

and then cancellations will occur in Eq. (9).

Designating by R_t the matrix of rotation over the time t with angular velocity w around the axis of the spiral, we obtain

$$(x,\ y,\ z)\, R_t = (x\cos wt - y\sin wt,\ x\sin wt + y\cos wt,\ z),$$

$$R_t = \begin{pmatrix} \cos wt & \sin wt & 0 \\ -\sin wt & \cos wt & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

It is obvious that R_t is the Jacobian of this transformation.

We shall say that the motion is steady if

$$\overline{W}(\overline{\chi}R_t,\ t) = \overline{W}(\overline{\chi},\ 0)\, R_t. \tag{13}$$

An example of steady motion is $\overline{V}_L$. For this reason, the motions $\overline{V}$ and $\overline{W}$ are simultaneously both steady or not.

We rewrite the identity (13) characterizing steady motion:

$$\overline{W}(\overline{\chi},\ \ t) = \overline{W}(\overline{\chi}R_{-t},\ 0) = \overline{W}_0(\overline{\chi}R_{-t})\, R_t. \tag{14}$$

Before substituting Eq. (14) into the system (9)-(12), we performed the following calculations:

$$\frac{\partial \overline{W}}{\partial t}(\overline{\chi},\ t) = \frac{\partial}{\partial t}\overline{W}_0(\overline{\chi}R_{-t})\, R_t = \overline{\chi}\left(\frac{d}{dt}\ R_{-t}\right)(\overline{\nabla}\,\overline{W}_0)(\overline{\chi}R_{-t})\, R_t + \overline{W}_0(\overline{\chi}R_{-t})\left(\frac{d}{dt}R_t\right),$$

$$(\overline{\nabla}\,\overline{W})^j_i(\overline{\chi},\ t) = \frac{\partial}{\partial\chi^i}\ W^j(\overline{\chi},\ t) = \frac{\partial}{\partial\chi^i}\Big[\sum_K W_0^K(\overline{\chi}R_{-t})\ (R_t)^j_K\Big] =$$

$$= \sum_{S,K} (R_{-t})_i^S\ (\bar{\nabla}\bar{W}_0)_S^K\,(\bar{\chi}R_{-t})\,(R_t)_K^j = (R_{-t}\,(\bar{\nabla}\bar{W}_0)\,(\bar{\chi}R_{-t})\,R_t)_i^j\,,$$

$$B(\bar{W})(\bar{\chi},\ t) = \bar{\nabla}\bar{W}(\bar{\chi},\ t) + (\bar{\nabla}\bar{W})^{\mathrm{T}}(\bar{\chi},\ t) =$$

$$= R_{-t}(\bar{\nabla}\bar{W}_0)(\bar{\chi}R_{-t})\,R_t + [R_{-t}(\bar{\nabla}\bar{W}_0)(\bar{\chi}R_{-t})\,R_t]^{\mathrm{T}} = R_{-t}\,B(\bar{W}_0)(\bar{\chi}R_{-t})\,R_t.$$

Here we employed the identities $(AB)^T = B^TA^T$, $(R_t)^T = R_{-t}$, $(R_{-t})^T = R_t$. Hence

$$B^2(\bar{W})(\bar{\chi},\ t) = R_{-t}B^2(\bar{W}_0)\,(\bar{\chi}R_{-t})\,R_t,$$

since $R_{-t}R_t = I_2$, and

$$I_2(\bar{W})(\bar{\chi},\ t) = I_2(\bar{W}_0)(\bar{\chi}R_{-t}),$$

since $S_pR_{-t}AR_t = S_pAR_tR_{-t} = S_pA$. We also calculate the following:

$$[\nabla^{\mathrm{T}} C(\bar{W})(\bar{\chi},\ t)]^j = \sum_i \frac{\partial}{\partial\chi^i}\,[\varphi_1 B_i^j + \varphi_2 (B^2)_i^j\,](\bar{\chi},\ t) =$$

$$= \sum_{i,K} (R_{-t})_i^K \frac{\partial}{\partial y^K}\,[\varphi_1(I_2(\bar{W}_0))(y)(R_{-t}B(\bar{W}_0)\,R_t)_i^j\,(y) +$$

$$+ \varphi_2(I_2(\bar{W}_0))\,(y)\,(R_{-t}\,B^2(\bar{W}_0)\,R_t)_i^j\,(y)]\Big|_{\bar{y}=\bar{\chi}R_{-t}} =$$

$$= \sum_{i,K,S,q} (R_{-t})_i^K\,(R_{-t})_i^S \frac{\partial}{\partial y^K}\,[\varphi_1\,(I_2\,(\bar{W}_0))\,B\,(\bar{W}_0) + \varphi_2\,(I_2\,(\bar{W}_0))B^2\,(\bar{W}_0)]_S^q(y)\,\times$$

$$\times\,(R_t)_q^j = \sum_{K,S,q} \delta_S^K \frac{\partial}{\partial y^K}\,[C\,(\bar{W}_0)\,(y)]_S^q\,(R_t)_q^j = [\nabla^{\mathrm{T}}\,C\,(\bar{W}_0)]\,(\bar{\chi}R_{-t})\,R_t.$$

Here we used the identity

$$\sum_i (R_{-t})_i^K\,(R_{-t})_i^S = \delta_S^K = \begin{cases} 1, & K = S, \\ 0, & K \neq S, \end{cases}$$

which is equivalent to the identity $R_{-t}R_t = I_2$.

Since $\bar{V}_L(\bar{\chi},\ t) = \bar{V}_L(\bar{\chi}R_{-t})R_t$, just as for $\bar{W}$, we have $(\bar{\nabla}\bar{V}_L) \times (\bar{\chi},\ t) = R_{-t}(\bar{\nabla}\bar{V}_L)(\bar{\chi}R_{-t})R_t$.

Substituting Eq. (14) and all obtained expressions into the system (9)-(12), we obtain

$$\rho\left[\left(\bar{\chi}\frac{d}{dt}\,R_{-t}\right)(\bar{\nabla}\bar{W}_0)\,(\bar{\chi}R_{-t})\,R_t + \bar{W}_0\,(\bar{\chi}R_{-t})\frac{d}{dt}\,R_t + \right.$$

$$+ \bar{W}_0\,(\bar{\chi}R_{-t})\,(\bar{\nabla}\bar{W}_0)\,(\bar{\chi}R_{-t})\,R_t + \bar{V}_L\,(\bar{\chi}R_{-t}) + \bar{W}_0)(\bar{\chi}R_{-t})\,(\bar{\nabla}\,\bar{V}_L)(\bar{\chi}R_{-t})\,R_t + \tag{15}$$

$$\left. + \bar{V}_L\,(\bar{\chi}R_{-t})\,(\bar{\nabla}\bar{W}_0)(\bar{\chi}R_{-t})\,R_t\right] - \nabla^{\mathrm{T}}\,P_2\,(\bar{\chi}R_{-t})\,R_t +$$

$$+ \nabla^{\mathrm{T}}\,C\,(\bar{W}_0)\,(\bar{\chi}R_{-t})\,R_t = 0,$$

$$S_p\,(\bar{\nabla}\bar{W}_0)\,(\bar{\chi}R_{-t}) = 0, \tag{16}$$

$$-\lambda\,(I_2\,(\bar{W}_0))\,(\bar{W}_0 + \bar{V}_L)\,(\bar{\chi}R_{-t})\,R_t = pr_{N_K^{\perp}}\bar{N}_K C\,(\bar{W}_0)R_t|_{K_0}, \tag{17}$$

$$-\lambda\,(I_2\,(\bar{W}_0))\bar{W}_0\,(\bar{\chi}R_{-t})\,R_t = pr_{N_L^{\perp}}\bar{N}_L\,C\,(\bar{W}_0)\,R_t|_{L_0}. \tag{18}$$

Here we employed the fact that $\bar{N}_K(\bar{\chi},\ t) = \bar{N}_K(\bar{\chi}R_{-t})R_t$, $\bar{N}_L(\bar{\chi},\ t) = \bar{N}_L(\bar{\chi}R_{-t})R_t$.

We denote $\bar{\chi}R_{-t}$ by y, right-multiply all equations by R_{-t}, and make the transformation in the first two terms of Eq. (15):

$$\rho\left[\left(\bar{\chi}\frac{d}{dt}\,R_{-t}\right)(\bar{\nabla}\bar{W}_0)\,R_t + \bar{W}_0\frac{d}{dt}\,Rt\right] = \rho\left[\left(\bar{\chi}R_{-t}R_t\frac{d}{dt}\,R_{-t}R_t\right)\times\right.$$

$$\times\,R_{-t}\bar{\nabla}\,\bar{W}_0R_t + \bar{W}_0\frac{d}{dt}\,R_tR_{-t}R_t = \rho\left[y\left(R_t\frac{d}{dt}\,R_{-t}\right)\bar{\nabla}\,\bar{W}_0R_t + \bar{W}_0\left(\frac{d}{dt}\,R_tR_{-t}\right)R_t\right],$$

$$R_t \frac{d}{dt} R_{-t} = \begin{pmatrix} \cos wt & \sin wt & 0 \\ -\sin wt & \cos wt & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -w \sin wt & -w\cos wt & 0 \\ w \cos wt & -w \sin wt & 0 \\ 0 & 0 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & -w & 0 \\ w & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = A,$$

$$\frac{d}{dt} R_t R_{-t} = \begin{pmatrix} -w \sin wt & w \cos wt & 0 \\ -w \cos wt & -w \sin wt & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \cos wt & -\sin wt & 0 \\ \sin wt & \cos wt & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & w & 0 \\ -w & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -A = A^{\mathrm{T}},$$

$$\overline{\Delta}_L(y) = -(wy^2,\ wy^1,\ y^3) = (y^1,\ y^2,\ y^3) \begin{pmatrix} 0 & w & 0 \\ -w & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -yA,$$

$$\overline{\nabla}\,\overline{V}_L = \begin{pmatrix} 0 & w & 0 \\ -w & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -A.$$

Then Eq. (15) assumes the form

$$\rho\,[yA(\overline{\nabla}\,\overline{W}_0)(y) - \overline{W}_0(y)A + \overline{W}_0(y)(\overline{\nabla}\,\overline{W}_0)(y) - yA(\overline{\nabla}\,\overline{W}_0)(y) - \\ - \overline{W}_0(y)A] - (\nabla^{\mathrm{T}} P_2)(y) + \nabla^{\mathrm{T}} C(\overline{W}_0)(y) = 0. \tag{19}$$

Finally, after cancellations we obtain the following system of equations:

$$\rho\,[-2\overline{W}_0(y)A + \overline{W}_0(y)(\overline{\nabla}\,\overline{W}_0)(y)] - (\nabla^{\mathrm{T}} P_2)(y) + \nabla^{\mathrm{T}} C(\overline{W}_0)(y) = 0, \tag{20}$$

$$S_p(\overline{\nabla}\,\overline{W}_0) = 0,\ \ y \in D_0, \tag{21}$$

$$-\lambda(I_2(\overline{W}_0))(\overline{W}_0 + \overline{V}_L) = pr_{N_K^\perp} \overline{N}_K C(\overline{W}_0)|_{K_0}, \tag{22}$$

$$-\lambda(I_2(\overline{W}_0))\overline{W}_0 = pr_{N_L^\perp} \overline{N}_L C(\overline{W}_0)|_{L_0}. \tag{23}$$

As one can see from the system of equations (20)-(23), Eq. (20) without the term $-2\rho\overline{W}_0(y)A$ would give a system of stationary equations of motion, which correspond to the inverse formulation (stationary screw and rotating casing). Equation (21), as we have already mentioned, is the equation of continuity. The boundary conditions (22)-(23) are the conditions of slipping on the wall, and in addition the presence of the term $+\overline{V}_L$ in the condition (22) means that the jacket rotates with angular velocity $-w$.

From the physical standpoint the term $-\rho\overline{W}_0(y)A$ is the Coriolis force, which appears on transforming from one formulation of the problem to another. It is possible that under some conditions this term can be neglected, as done in [1-3]. A quantitative estimate of the magnitude of the Coriolis force with respect to the viscosity and inertial forces can be obtained from the equations of motion by introducing some criterion, analogously to the introduction of the criterion Re. The quantity $K = 2\rho wHb/\varphi_1$ obtained in this manner will characterize the ratio of the Coriolis to viscous forces. Here H and b are the height and width of the spiral channel.

For some working media and, in particular, for a melt of polyethyleneterephthalate (lavsan) $\rho = 1380$ kg/m^3 and $\varphi_{10} = 9.3$ Pa·sec ($t^0 = 275°$C) and for real dimensions of a spiral channel with worm diameter D = 0.052 m and height H = 0.005 m with angular rotational velocity of the screw w = 1/C the criterion K = 0.25, which indicates that the Coriolis forces must be taken into account.

Thus the presence of the term $-2\rho\overline{W}_0(y)A$ in Eq. (20) shows that the situation "rotating screw" – the direct formulation – is not equivalent to the situation "rotating casing" – the inverse formulation.

NOTATION

Here x, y, and z are running coordinates; $\Psi(x, y, z)$ is the surface of constant level; D_t is the region occupied by the liquid at the time t; K_t is the part of the boundary D_t

belonging to the casing of the screw; L_t is the part of the boundary D_t belonging to the screw; $\bar{\chi}(x, y, z)$ is a point in the space; $\bar{V}(\bar{\chi}, t)$ is the flow velocity vector at the point $\bar{\chi}$ at the time t; $\varphi_1(I_2)$, $\varphi_2(I_2)$ are the material functions of a Reiner–Rivlin liquid and characterize the effective and transverse viscosity; B(V) is the first White–Metzner cinematic tensor (strain rate tensor); P is the pressure; $I_2(\bar{V})$ is the second invariant of the strain rate tensor; ρ is a constant of the liquid; $\bar{N}_K(\bar{\chi})$ is the unit inner normal vector to K_t at the point $\bar{\chi} \in D_t$; $\bar{N}_L(\bar{\chi})$ is the unit outer normal vector to L_t at the point $\bar{\chi} \in L_t$; $pr_N^{\perp}$ is the orthogonal projection on a surface perpendicular to N; w is the angular rotational velocity of the screw; T designates transposition; and R_t is the matrix of rotation over a time t with angular velocity w around the axis of the spiral.

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SMALL-ASPECT TOMOGRAPHY OF HEATED FLOWS BASED ON IR-RADIOMETRIC MEASUREMENTS

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An algorithm for performing tomographic analysis of nonuniform heated gas flows is proposed. The algorithm employs infrared radiometric measurements and takes into account the real line structure of the vibrational–rotational bands of gases, including reabsorption.

The radiation emitted from a flow of heated gases contains rich information about the internal thermodynamic properties of the flow. It is natural to develop optical methods of diagnostics of such flows, especially since they have a number of significant advantages, including the fact that the diagnostics is performed remotely and does not disturb the medium under study. Maximum intensity of equilibrium thermal emission at a temperature of the order of 1000 K lies in the infrared region of the spectrum. Vibrational–rotational bands of many molecular gases lie in the same region. For this reason, in order to determine the temperature and concentration of the emitting components it is best to employ measurements in the IR region of the spectrum.

Methods for performing diagnostics of a uniform layer, which are based on measurements of the absorption coefficient and brightness of the radiation emitted by the layer in different spectral regions, are described in a number of works [1-3]. However, they are not applicable for diagnostics of flows which have significant spatial nonuniformity. The methods of computer tomography are widely employed to investigate nonuniform spatial struc-

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